Multiple-scales analysis of plasma response to intense laser field and plasma excitation

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(Received 6 May 1996; revised manuscript received 16 September 1996)

A method of multiple scales is used to analyze a nonequilibrium distribution of the electron plasma in an intense laser field. A dielectric response function, which is nonlinear for the laser field, with wave vectors at arbitrary angles to the laser field is presented by using an appropriate small fluctuation of slowly varying quantities to linearize the kinetic equation of plasma. The numerical results for dielectric properties of plasma in various directions are displayed. The plasma instability in high electric field reported by Morawetz and Jauho [Phys. Rev. E **50**, 474 (1994)] does not exist under any intensity laser fields. The most excited wave modes are damped even though the imaginary part of the dielectric function becomes negative in certain conditions. For the limit of the parameter $k\lambda_D \ll 1$, a corrected analytical expression of dispersion relation is obtained. This modification of the excited wave dispersion relation results in a great change of the stimulated Raman scattering instability, particularly for the large-angle sideward Raman instability. The screening properties of plasma under the intense laser field and the dependence of wave mode excitation on plasma parameters and angles of the wave vector from the laser field are also discussed. [S1063-651X(97)07802-1]

PACS number(s): 52.40.Nk, 52.35.Mw

I. INTRODUCTION

The study of the interaction of intense laser light with plasmas serves as an excellent introduction to the field of plasma physics. Both the linear and nonlinear theories of plasma waves, instabilities, and wave-particle interactions are important for understanding the laser plasma coupling. In particular, recent advances in short-pulse laser technology [1–3] ($P \approx 10^{18}$ W/cm^2, $\tau_L \approx 2\pi\omega_p^{-1} \sim 1$ psec) make the nonlinear laser-matter interactions possible. Such a high-intensity laser field, for example, leads to a number of different laser-plasma and laser–electron-beam interaction phenomena [4–12]. For the different intensities the laser field irradiates onto the plasma, various nonlinear properties will be produced, where the wave mode excitation of the plasma, however, is a collective response of the plasma coupling with the external field. In particular, the plasma nonlinear coupling with incident laser light results in a scattering light wave and an electron plasma wave [stimulated Raman scattering (SRS)]. It has been actively investigated theoretically [13–15] and experimentally [16,17] for nearly 20 years because this instability is an important issue in laser inertial confinement fusion [18], x-ray laser [19,20], and laser wakefield generation [21,22], and so on. In a three-wave (incident laser light, scattering light, and plasma waves) interaction, the behaviors of the plasma wave are significant for understanding various instabilities. As a basis of these currently active issues, the investigations of plasmas phenomena in a strong oscillating electromagnetic field are of great significance. More recently, Morawets and Jauho [23] indicated that under a high static electric field a plasma instability exists for certain intensity electric fields and frequencies. Their results were obtained by using a generalized Kadanoff-Baym ansatz and a displaced Maxwellian distribution function.

In this paper, we show that this instability does not exist under an intense but constant laser field. The attractive strength values for incident laser light is of the subpicosecond order of short pulses. In order to connect this work with current relevant issues we consider such a short pulse laser as well, i.e., assuming that the laser pulse length τ_L is smaller than the inverse of the ion plasma frequency ω_{pi}. In this case, the motion of the ions can be neglected. The electron-electron collisions cannot equilibrate the distribution of the electron plasma sufficiently rapidly. Thus the electrons would not be in equilibrium. On the other hand, it is difficult to calculate the nonequilibrium distribution function of the electrons by solving the Fokker-Planck or Vlasov equation, and using the conventional methods to treat the perturbation is not valid due to the presence of an intense laser field. In our work a multiple-scale method is used to analyze the nonequilibrium distribution of the electron plasma in an intense laser field. By using an appropriate small fluctuation of slowly varying quantities, we linearize the kinetic equation of plasma self-consistently and obtain a dielectric response function and a nonequilibrium distribution function resulting from the electron quiver motion. A remarkable result, however, is that in all directions of the wave vector, the most excited wave modes are damped and at the wave vector zone where the imaginary part of the dielectric function is negative there are no zero points of the real part. The dispersion relation in the limits of $k\lambda_D \ll 1$ and screening properties of the plasma are changed under an intense laser field. However, these results lead to a large modification of the SRS temporal growth rate. Interestingly, for a certain strength laser field, the large-angle sidescatter is more important compared to backscatter around the quarter critical density. When the wave vector is parallel to the laser propagation direction these properties return to those of the Maxwellian distribution in the absence of an intense laser field and the SRS growth rate returns to the usual properties.

It is worth mentioning that many authors [11,24–26] worked in a frame where the electrons have only their random thermal motion and the oscillatory motion of the laser field is transferred to the ions. In such a frame their results appeared in the form of sums of Bessel functions. On the other hand, the Bessel function appears also in the discussions of the quickly varying behavior of plasma oscillation [27–29]. Here we will concentrate on the slowly varying behaviors of plasma oscillation and work in the laboratory frame where the Bessel function is not to emerge in the obtained results.

II. MULTIPLE-SCALE ANALYSIS OF KINETIC EQUATION

The basic equations describing the interaction of laser field with the plasma is the Vlasov equation

$$\frac{\partial f_j}{\partial t}+\mathbf{v}\cdot\frac{\partial f_j}{\partial \mathbf{x}}+\frac{q_j}{m_j}\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial f_j}{\partial \mathbf{v}}=0 \tag{1}$$

plus the Maxwell equations, where the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are conspicuously the sum of those applied from outside sources and those induced from the internal particle distributions, i.e, $\mathbf{E}=\mathbf{E}_{\rm ext}+\mathbf{E}_{\rm ind}$ and $\mathbf{B}=\mathbf{B}_{\rm ext}+\mathbf{B}_{\rm ind}$, $f_j=f_j(\mathbf{x},\mathbf{v};t)$ is the distribution function of the jth-type particles, in phase space $(\mathbf{x},\mathbf{v})$, and q_j and m_j are the charge and mass of the jth-type particles, respectively. Our analysis starts from Eq. (1). We write $\mathbf{x}$ and t in multiple-scale forms

$$T_0=t,\quad T_1=\epsilon t,\quad T_2=\epsilon^2 t,\ \ldots \tag{2a}$$

and

$$\mathbf{x}_0=\mathbf{x},\quad \mathbf{x}_1=\epsilon\mathbf{x},\quad \mathbf{x}_2=\epsilon^2\mathbf{x},\ \ldots, \tag{2b}$$

where ϵ is a small parameter, T_0 and $\mathbf{x}_0$ represent fast scales, T_1 and $\mathbf{x}_1$ represent slower scales, T_2 and $\mathbf{x}_2$ represent more slower scales, and so on. In our physical problem T_0 and $\mathbf{x}_0$ correspond to the levels of laser frequency ω_L and wave vector $c\mathbf{k}_L/v_{\rm th}$, where $v_{\rm th}$ is the thermal velocity of electrons and T_1 and $\mathbf{x}_1$ are the levels of the plasma frequency ω_p (where $\omega_p=4\pi e^2 n_j/m_j$, with n_j the ambient density of the jth-type particles), wave vector $\mathbf{k}$, and laser wave vector $\mathbf{k}_L$. Hence the distribution function can be written as

$$f_j(\mathbf{v},\mathbf{x};t)=f_j(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\ldots;T_0,T_1,T_2,\ldots).$$

Instead of determining f_j as a function of $(\mathbf{v},\mathbf{x};t)$, we determine f_j as a function of $(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\ldots;T_0,T_1,T_2,\ldots)$. To this end, we change the independent variable in the original equation (1) from $(\mathbf{v},\mathbf{x};t)$ to $(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\ldots;T_0,T_1,T_2,\ldots)$. Using a chain rule, we have

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial T_0}+\epsilon\frac{\partial}{\partial T_1}+\epsilon^2\frac{\partial}{\partial T_2}+\cdots, \tag{3a}$$

$$\frac{\partial}{\partial \mathbf{x}}=\frac{\partial}{\partial \mathbf{x}_0}+\epsilon\frac{\partial}{\partial \mathbf{x}_1}+\epsilon^2\frac{\partial}{\partial \mathbf{x}_2}+\cdots. \tag{3b}$$

Hence, for the plasma electrons Eq. (1) becomes

$$\left(\frac{\partial}{\partial T_0}+\epsilon\frac{\partial}{\partial T_1}+\epsilon^2\frac{\partial}{\partial T_2}+\cdots\right)f_e+\mathbf{v}\cdot\left(\frac{\partial}{\partial \mathbf{x}_0}+\epsilon\frac{\partial}{\partial \mathbf{x}_1}+\epsilon^2\frac{\partial}{\partial \mathbf{x}_2}+\cdots\right)f_e-\frac{e}{m_e}\times\left[(\mathbf{E}_L+\epsilon\mathbf{E}_{\rm ind})+\frac{1}{c}\mathbf{v}\times\epsilon\mathbf{B}_{\rm ind}\right]\cdot\frac{\partial}{\partial \mathbf{v}}f_e=0, \tag{4}$$

where $\mathbf{E}_L$ is the laser electric field and a fast scale quantity and $\mathbf{E}_{\rm ind}$ and $\mathbf{B}_{\rm ind}$ are the induced fields in the plasma and two slow scale quantities. Our purpose is to seek a uniform approximate solution to Eq. (4) in the form

$$f_e=f_e^{(0)}(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\ldots;T_0,T_1,T_2,\ldots)+\epsilon f_e^{(1)}(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\ldots;T_0,T_1,T_2,\ldots)+\cdots. \tag{5}$$

Substituting for f_e from Eq. (4) and equating each of the cofficients ϵ^0 and ϵ to zero, we have

$$\frac{\partial}{\partial T_0}f_e^{(0)}+\left(\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}_0}\right)f_e^{(0)}-\frac{e}{m_e}\left(\mathbf{E}_L\cdot\frac{\partial}{\partial \mathbf{v}}\right)f_e^{(0)}=0, \tag{6}$$

$$\left[\frac{\partial}{\partial T_1}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}_1}-\frac{e}{m_e}\left(\mathbf{E}_{\rm ind}+\frac{1}{c}\mathbf{v}\times\mathbf{B}_{\rm ind}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right]f_e^{(0)}+\left(\frac{\partial}{\partial T_0}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}_0}-\frac{e}{m_e}\mathbf{E}_L\cdot\frac{\partial}{\partial \mathbf{v}}\right)f_e^{(1)}=0. \tag{7}$$

Equation (6) is a zeroth-order equation corresponding to a zeroth-order solution that gives a nonequilibrium distribution and Eq. (7) is a first-order equation corresponding to a first-order solution that is a correction to the zeroth-order solution. We require only to retain the equations to first order for our problems. In fact, Eq. (7) can describe both the quickly varying (wave mode frequencies are greater than or approximately equal to the light wave frequency) and the slowly varying (wave mode frequencies are far less than the light wave frequency) behaviors in the plasma oscillation extraordinarily well, which, of course, is the result of utilizing the method of multiple-scale analysis. If one neglects those two slowly varying terms, $\partial/\partial T_1 f_e^{(0)}$ and $\mathbf{v}\cdot\partial/\partial\mathbf{x}_1 f_e^{(0)}$, Eq. (7) gives a fast varying approximation just like Jackson done [27].

Solving Eq. (6) by setting the laser field $\mathbf{E}_L=\mathbf{E}_0\cos(\mathbf{k}_L\cdot\mathbf{x}-\omega_L t)$ in which the phase $\mathbf{k}_L\cdot\mathbf{x}-\omega_L t$ becomes $\mathbf{k}_L\cdot\mathbf{x}_0-\omega_L T_0$ in our consideration of multiple scales because $k_L\ll\omega_L/v_{\rm th}$, the term $\mathbf{k}_L\cdot\mathbf{x}_0$ in the phase may be treated as a constant. Consequently, we obtain the form of the zeroth-order solution of the kinetic equation (1),

$$f_e^{(0)}\sim\exp\left[-\frac{m_e}{2T_e}(\mathbf{v}-\mathbf{v}_E)^2\right], \tag{8}$$

where it has been assumed that the distribution of the plasma electron is Maxwellian before the laser light entered into the plasma and $\mathbf{v}_E=(e\mathbf{E}_0/m_e\omega_L)\sin(\mathbf{k}_L\cdot\mathbf{x}_0-\omega_L T_0)$ is the quiver velocity of the electrons in the laser field. Hence let

$$f_e^{(0)}=A(T_1;\mathbf{x}_1,\mathbf{v})\exp\left[-\frac{m_e}{2T_e}(\mathbf{v}-\mathbf{v}_E)^2\right]\equiv Ag_0. \quad (9)$$

We note that A is not a constant but a function of the slow scales T_1, $\mathbf{v}$, and $\mathbf{x}_1$ because $f_e^{(0)}$ is a function of $(\mathbf{v},\mathbf{x}_0,\mathbf{x}_1,\ldots;T_0,T_1,\ldots)$ and the derivatives in Eq. (7) are with respect to $(\mathbf{x}_1;T_1)$, which is not known at this level of approximation; it is determined at subsequent levels of approximation by eliminating the secular terms. It is remarkable that we are only interested in the zeroth-order solution $f_e^{(0)}$ because our final goal is to find the dielectric function that can be obtained from the first-order equation (7) by using a self-consistent linearizing method. The above equations are, however, kept to first-order. On the other hand, the slow scale cofficient A is dependent on the characteristic parameters of the plasma, which differ from these of Ref. [23]. It may be determined in a self-consistent linearizing approximation.

For expression (9), actually, it is very clear in the physical picture that the electron thermal motion is superposed a quiver component due to the presence of an external laser field, which is a drift Maxwellian distribution. Because the fast scales $\mathbf{x}_0$ and T_0 are included in v_E and $\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}_E)<0$, Eq. (9) reflects a coupling motion of both fast and slow motions.

III. DIELECTRIC RESPONSE FUNCTION OF PLASMA

In principle, the first-order scale equation contains terms that produce secular terms. For a uniform expansion, these terms must be eliminated. From Eqs. (6) and (7) we see that the term including $f_e^{(0)}$ in Eq. (7) averaging over T_0, which will contribute to a term proportional to T_0 for $f_e^{(1)}$, is a secular term and should be set equal to zero, so that

$$\left[\frac{\partial}{\partial T_1}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}_1}-\frac{e}{m_e}\left(\mathbf{E}_{\rm ind}+\frac{1}{c}\mathbf{v}\times\mathbf{B}_{\rm ind}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right]A\langle g_0\rangle=0. \quad (10)$$

It can be seen that in the treatment of the secular term only the wave frequencies that are far less than the laser frequency are left. Obviously, the treatment is identical to the considerations of the current subjects of interest. For the slowly varying behaviors in the plasma oscillation, Eq. (10) is a highly appropriate description.

Assume that these slowly varying quantities have perturbations

$$\delta\mathbf{E}_{\rm ind}\exp[i(\mathbf{k}\cdot\mathbf{x}_1-\omega T_1)]+\text{c.c.}, \quad (11a)$$

$$\delta\mathbf{B}_{\rm ind}\exp[i(\mathbf{k}\cdot\mathbf{x}_1-\omega T_1)]+\text{c.c.} = \frac{c}{\omega}(\mathbf{k}\times\delta\mathbf{E}_{\rm ind})\exp[i(\mathbf{k}\cdot\mathbf{x}_1-\omega T_1)]+\text{c.c.}, \quad (11b)$$

$$A_0+\delta A(\mathbf{v})\exp[i(\mathbf{k}\cdot\mathbf{x}_1-\omega T_1)]+\text{c.c.}, \quad (11c)$$

respectively, where $A_0=(m_e/2\pi T_e)^{3/2}$ is the normalized constant of Maxwellian distribution, which is a fundamental requirement satisfying the initial condition. Substituting Eqs. (11a)–(11c) into Eq. (10) and then linearizing, we obtain

$$\delta A\langle g_0\rangle(-i\omega+i\mathbf{k}\cdot\mathbf{v}) = \frac{e}{m_e}A_0\left\langle\frac{\partial g_0}{\partial \mathbf{v}}\right\rangle\cdot[(1-\mathbf{k}\cdot\mathbf{v}/\omega)\tilde{\mathbf{I}}+\mathbf{v}\mathbf{v}/\omega]\cdot\delta\mathbf{E}_{\rm ind}, \quad (12)$$

where $\tilde{\mathbf{I}}$ is a unit tensor and angular brackets represent an average over a period associated with the frequency of laser. Therefore, the amplitude $\delta\mathbf{J}$ of the induced current produced by field disturbance can be written as

$$\delta\mathbf{J}=-en_e\int_{-\infty}^{\infty}\mathbf{v}\delta A\langle g_0\rangle d\mathbf{v}, \quad (13)$$

where n_e is the density of the electron plasma. Substituting Eqs. (11a)–(11c) into Eq. (12), according to the complex Fourier forms of Maxwell's equations, and utilizing Eq. (13), finally, the dielectric tensor function is obtained

$$\tilde{\mathbf{D}}(\mathbf{k},\omega)=\left(1-\frac{\omega_{pe}^2}{\omega^2}\right)\tilde{\mathbf{I}}-A_0\frac{\omega_{pe}^2}{\omega^2}\int_{-\infty}^{\infty}d\mathbf{v}\frac{\mathbf{v}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v}-\omega}\mathbf{k}\cdot\left\langle\frac{\partial g_0}{\partial \mathbf{v}}\right\rangle, \quad (14)$$

where $\omega_{pe}=(4\pi e^2n_e/m_e)^{1/2}$ is the frequency of the electron plasma.

Making use of the conventional methods [30] and averaging in the wave vector direction $\mathbf{k}\cdot\tilde{\mathbf{D}}(\mathbf{k},\omega)\cdot\mathbf{k}/k^2$, the dielectric response function can be calculated as

$$D(\mathbf{k},\omega)=1+\frac{k_D^2}{k^2}\langle W(\xi)\rangle, \quad (15)$$

where the dispersion function

$$W(\xi)=1-\xi\exp(-\xi^2/2)\int_0^{\xi}\exp(t^2/2)dt + i(\pi/2)^{1/2}\xi\exp(-\xi^2/2) \quad (16)$$

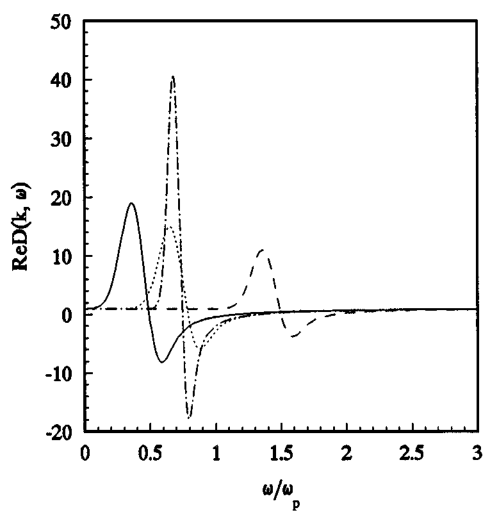

FIG. 1. Real part of the dielectric response function vs the frequency for the different parameters: $k\lambda_D=0.05$, $v_{\rm os}/v_{\rm th}=10.0$, $\theta=0.0$ (dash-dotted line); $k\lambda_D=0.1$, $v_{\rm os}/v_{\rm th}=10.0$, $\theta=0.0$ (dashed line); $k\lambda_D=0.1$, $v_{\rm os}/v_{\rm th}=5.0$, $\theta=0.0$ (dotted line); and $k\lambda_D=0.1$, $v_{\rm os}/v_{\rm th}=5.0$, $\theta=0.3\pi$ (solid line), respectively.

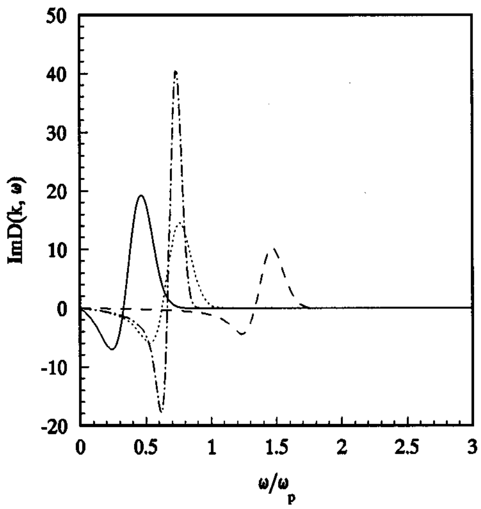

FIG. 2. Imaginary part of the dielectric function vs the frequency. The parameters are the same as in Fig. 1.

and

$$\xi = \frac{v_p}{v_{\rm th}} - \frac{v_{\rm os}}{v_{\rm th}} \cos\theta \sin\psi. \tag{17}$$

$v_{\rm os} = eE_0/m_e\omega_L$ and $v_{\rm th} = (T_e/m_e)^{1/2}$ are the amplitude of the quiver velocity of electrons in the laser field and the thermal velocity of electrons, respectively, $k_D = (4\pi e^2 n_e/T_e)^{1/2} = \lambda_D^{-1}$ is the Debye wave number, θ is an angle of the wave vector and laser field vector, $v_p = \omega/k$ is the phase velocity of the wave, and ψ is the phase of the laser field. The second term on the right-hand side of Eq. (16) involves an orthostatic probability integration that can be represented by a special function $\xi^2 {}_1F^1(1;\frac{3}{2};-\frac{1}{2}\xi^2)$, where ${}_1F^1(\alpha;\beta;z)$ is known as the Kummer function.

It is worth noting that if one seeks the usual electrostatic form rather than that of Eq. (14), one can usefully write the denominator of the dielectric function as $\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}_E) - k\xi v_{\rm th}$ and change the integration variable from $\mathbf{v}$ to $(\mathbf{v}-\mathbf{v}_E)$, since the different feature in the electron dielectric function is the presence of $\mathbf{v}_E$ in the oscillating Maxwellian. Hence one now can obtain the standard result in the usual way [30], but with the usual $\omega/kv_{\rm th}$ replaced by ξ, then to be followed by averaging over the oscillation phase. Note that this is the reverse of the usual procedure in the calculation for the cyclotron orbits of a hot magnetoplasma [12] or for the oscillation orbits in previous oscillating-electron plasmas [11,31–33]. This procedure gives the result of Eq. (15). In addition, it is obvious that the response function $D(\mathbf{k},\omega)$, when $\theta = \pm\pi/2$, returns to that of the Maxwellian distribution in the absence of an external laser field. We also can see, from Eqs. (24)–(27), that the laser field modifications to the dielectric properties of the plasma embody the function ξ completely because $\mathbf{k}$ and ω appear in ξ only, and in the complex ω plane, the position of the poles shifts in comparison to the Maxwellian. The detailed analyses will be presented numerically in the next section.

IV. NUMERICAL RESULTS OF MODE EXCITATION

We now consider the real part of the dielectric function Eq. (15). The real part Re$D(\mathbf{k},\omega)$ as a function of the frequency ω is plotted for different parameters. Figure 1 displays the variation of Re$D(\mathbf{k},\omega)$ for different parameters: $v_{\rm os}/v_{\rm th}$, $k\lambda_D$, and the angle θ, respectively. For a certain temperature of the plasma the exciting frequencies move to higher values of ω when both the strength of the laser field and the parameter $k\lambda_D$ are increased, respectively. However, increasing the angle θ, the exciting frequencies move to lower values of ω. The amplitude of Re$D(\mathbf{k},\omega)$ is susceptible to the parameter $k\lambda_D$. The smaller the parameter $k\lambda_D$, the larger the amplitude. It can be seen from Fig. 3 that the exciting modes result in a narrow region of the $k\lambda_D$ value. In the direction of the wave vector $\mathbf{k}$ parallel to the laser field, no excitations exist for $k\lambda_D$ exceeding 0.3 in the case of arbitrary laser field strength values, and the higher the strength, the narrower the region of the $k\lambda_D$ value where the excitation occurs. That is to say, when $k\lambda_D \geqslant 0.3$, the Landau damping typically becomes important, which is identical to the theoretical analysis of Ref. [34].

Figure 2 shows the imaginary part of the dielectric function Im$D(\mathbf{k},\omega)$ as a function of the frequency ω. Im $D(\mathbf{k},\omega)$ becomes negative in a certain region of frequency. However, combining Fig. 1 with Fig. 2, one can see that this negative imaginary part does not involve the zero points of the real part in the dielectric function, which implies that it

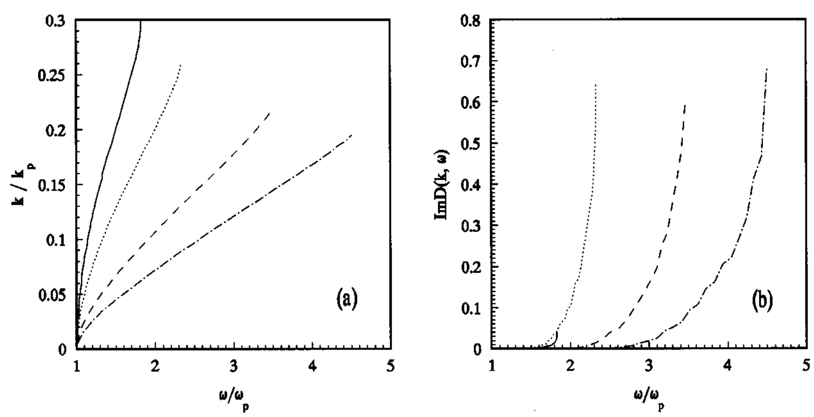

FIG. 3. (a) Relationship of ω and k for the second excitation point. The corresponding parameters are $v_{\rm os}/v_{\rm th} = 15.0$, $\theta = 0.0$ (dash-dotted line); $v_{\rm os}/v_{\rm th} = 10.0$, $\theta = 0.0$ (dashed line); $v_{\rm os}/v_{\rm th} = 5.0$, $\theta = 0.0$ (dotted line); and $v_{\rm os}/v_{\rm th} = 5.0$, $\theta = 0.3\pi$ (solid line). (b) Imaginary part of the dielectric function vs the values ω and k, which correspond to the second excitation point. The parameters are the same as in (a).

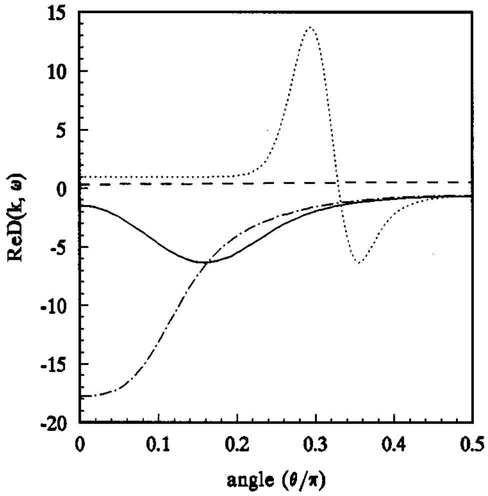

FIG. 4. Real part of the dielectric function vs the angle θ for $k\lambda_D=0.05$, $\omega=0.8\omega_p$, $v_{\rm os}/v_{\rm th}=10.0$ (dash-dotted line); $k\lambda_D=0.05$, $\omega=1.5\omega_p$, $v_{\rm os}/v_{\rm th}=10.0$ (dashed line); $k\lambda_D=0.1$, $\omega=0.8\omega_p$, $v_{\rm os}/v_{\rm th}=10.0$ (dotted line); and $k\lambda_D=0.1$, $\omega=0.8\omega_p$, $v_{\rm os}/v_{\rm th}=5.0$ (solid line).

does not follow the instability of the plasma wave. The first exciting point corresponds to a very large positive imaginary part, which represents a large Landau damping. Therefore, a possible exciting mode exists only at the second zero point of the real part, which can be seen from Figs. 3(a) and 3(b). In order to understand these excitation properties in more detail the real part of the dielectric function as a function of the angle θ and strength value ($v_{\rm os}/v_{\rm th}$) are displayed in Figs. 4 and 5 respectively. It is noteworthy that in the different directions the exciting properties of the plasma are distinctive, which differs from the usual situation. From the above discussions we find that the nonlinear response properties of the plasma to an intense laser field concentrate on the influence of the laser field on the distribution of the plasma. Essentially, in the contour integration of expression

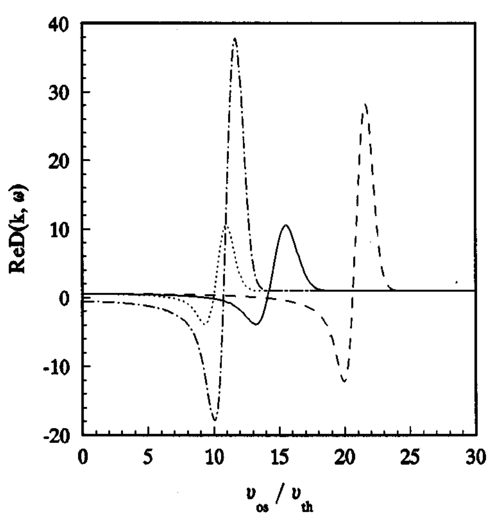

FIG. 5. Real part of the dielectric function vs the laser strengths $v_{\rm os}/v_{\rm th}$. The parameters are the same as in Fig. 4.

(14) the distribution of poles in the complex ω plane have displaced due to the presence of a laser field. The space distribution of these poles determines the response of the plasma on an external field. The discussed properties exist just because the poles have made a redistribution.

V. DISPERSION RELATION AND SHIELDING PROPERTIES

Now we consider the case of the parameter $k\lambda_D \ll v_{\rm th}/v_{\rm os}<1$, which means $\xi \gg 1$. In this limit the real part of Eq. (16) can be expressed in a convergent series

$$[W(\xi)]_R=-\xi^{-2}-3\xi^{-4}-\cdots-[(2n-1)!!]\xi^{-2n}-\cdots, \tag{18}$$

where $(2n-1)!!=(2n-1)\times(2n-3)\times\cdots\times 3\times 1$. Taking the first two terms on the right-hand side of Eq. (18) and then averaging in a laser cycle yields

$$\begin{aligned}\langle W(\xi)\rangle_R \simeq & -(2\pi)^{-1}\left(k\lambda_D\frac{\omega_{pe}}{\omega}\right)^2\int_0^{2\pi}d\psi(1-2\beta\sin\psi \\ & +3\beta^2\sin^2\psi)-3(2\pi)^{-1}\left(k\lambda_D\frac{\omega_{pe}}{\omega}\right)^4 \\ & \times\int_0^{2\pi}d\psi(1-4\beta\sin\psi+10\beta^2\sin^2\psi),\end{aligned} \tag{19}$$

where $\beta=(k\lambda_D\omega_{pe}/\omega)(v_{\rm os}/v_{\rm th})\cos\theta$. Substituting Eq. (19) into Eq. (15), setting $D(\mathbf{k},\omega)=0$, and neglecting some high-order small terms, finally, we obtain an analytical expression of dispersion relation

$$\omega^2=\omega_{pe}{}^2\left[1+3k^2\lambda_D{}^2+\frac{3}{2}k^2\lambda_D{}^2\left(\frac{v_{\rm os}}{v_{\rm th}}\right)^2\cos^2\theta\right]. \tag{20}$$

In the process of the derivation above we have neglected relevant terms higher than third-order of small quantities. Obviously, the excited wave frequency increases substantially and in different directions it is distinctive. If one neglects the $v_{\rm os}$-dependent term, Eq. (20) recovers the Bohm-Gross frequency. The third term on the right-hand side of Eq. (20) is called the laser field modified term. This term becomes zero when $\theta=\pi/2$. It is to see that the external laser field does not affect the plasma oscillation along the direction of laser propagation. Evidently, from Eq. (20), for a high field ($v_{\rm os}/v_{\rm th}>1$) the modified term is larger than the second term in the transverse direction (wave vector parallel to laser field vector), so this term is significant to the transverse plasma oscillation along the direction of the laser field. Combining Eq. (20) with Figs. 3(a) and 3(b) we can see that, for the cold plasma, the maximum of the correction term is allowed to arrive at $\sqrt{8}\omega_{pe}$ in the region of the exciting waves because the wave modes of $\omega/\omega_{pe}\leq 3$ can be excited for a certain strength value of the laser.

In order to understand the anisotropic dielectric properties of plasma further, we consider the limit of $\omega=0$. One can find, from Eqs. (15)–(17), that the dispersion function $\langle W(\omega=0)\rangle \neq 1$ for $\theta\neq\pi/2$ in this limit. Therefore, the screening property of the plasma is changed due to an intense laser field. The screening length becomes

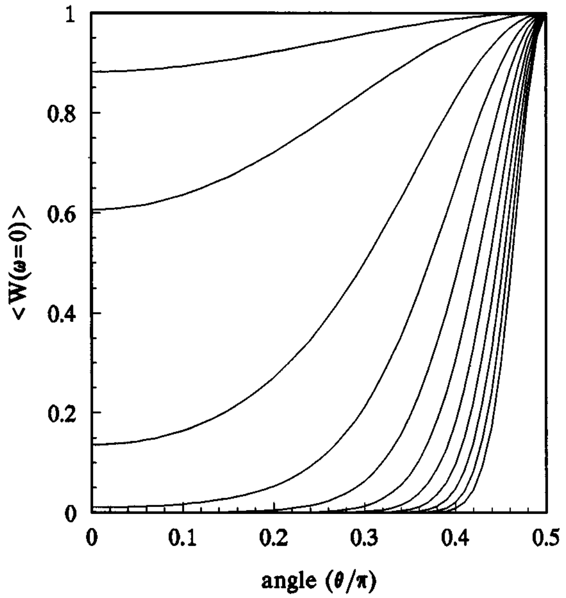

FIG. 6. Dispersion function at $\omega=0.0$ vs the angle θ. The corresponding laser field strengths of the curves are $v_{\rm os}/v_{\rm th}=0.5$, 1.0, 2.0, 3.0, ..., 10.0 from top to bottom.

$$r_D=[4\pi\langle W(\omega=0)\rangle n_e e^2/T_e]^{-1/2}=\lambda_D[\langle W(\omega=0)\rangle]^{-1/2} \tag{21}$$

and r_D is determined by not only T_e and n_e but also $v_{\rm os}$ and θ. The evolution of $\langle W(\omega=0)\rangle$ along with strength value of the laser $v_{\rm os}/v_{\rm th}$ and the angle θ are plotted, respectively, in Fig. 6. One can see that $\langle W(\omega=0)\rangle \leqslant 1$. The equality sign is satisfied only in two cases of $v_{\rm os}=0$ or $\theta=\pi/2$. The screening length increases when the strength of the laser is raised. When the strength value increases to $v_{\rm os}/v_{\rm th}\geqslant 4.0$, $\langle W(\omega=0)\rangle\approx 0$, which means that the screening effect disappears in the transverse direction. As the angle θ increases gradually, the screening length decreases correspondingly and returns to the Debye length when θ increases to $\pi/2$. We can see from Fig. 6 that the screening effect disappears within certain limits of θ when $v_{\rm os}/v_{\rm th}$ is sufficiently large. Furthermore, in the problem of Coulomb scattering, which is closely related to the screening of an effective potential $\sim\exp(-r/r_D)$, one sets the maximum impact parameter $b_{\max}$ equal to the screening length in order to avoid a logarithmic divergence, i.e., takes the Coulomb logarithm as $\ln\Lambda=\ln(12\pi n_e\lambda_D^3)$. From the above discussion one can find that the screening length λ_D becomes r_D, which is larger than λ_D. Under an intense laser field it will become very large and tends to infinity when $v_{\rm os}/v_{\rm th}$ is sufficiently large in the direction parallel or approximately parallel to the laser field. Of course, whether this shielding phenomenon is outstanding within the interaction time ω_{pe}^{-1} will be worth studying further in the future.

VI. RAMAN INSTABILITY

The stimulated Raman scattering instability arises from the decay of a pump photon into a scattered photon and a plasmon. What we are interested in here is the effects of the discussed wave exciting properties on the SRS growth rate. We look at the case of a steady-state interaction where the pump is continuous over all space (linearity approximation) and not too strong. In this case the temporal growth rate of the unstable waves for the backward and sideward SRS instabilities is given by [35]

$$\gamma=\frac{1}{4}kv_{\rm os}\left[\frac{\omega_{pe}^2}{\omega_{ek}(\omega_L-\omega_{ek})}\right]^{1/2}, \tag{22}$$

where $\omega_{ek}=(\omega_{pe}^2+3k^2v_{\rm th}^2)^{1/2}$ is the Bohm-Gross frequency and the wave number $\mathbf{k}$ is given by the dispersion relation $(\omega_{ek}-\omega_L)^2-c^2(\mathbf{k}-\mathbf{k}_L)^2-\omega_{pe}^2=0$. Since most experiments start out with a very low temperature for the electrons ($\sim$10 eV) one can assume that the initial electronic temperature is cold (i.e. $\omega_{pe}^2\gg 3k^2v_{\rm th}^2$). In this case the growth rate becomes

$$\gamma=0.5\omega_L\frac{v_{\rm os}}{c}\left(\frac{\omega_{pe}/\omega_L}{1-\omega_{pe}/\omega_L}\right)^{1/2}. \tag{23}$$

In our considerations the growth rate of instability can be changed because the dispersion relation of the electron plasma wave has been modified [see Eq. (20)]. From a direct derivation, the relevant density fluctuation equation that describes the instability now becomes

$$\left[\frac{\partial^2}{\partial t^2}+\omega_{pe}^2-3v_{\rm th}^2\left(1+\frac{1}{2}\mu^2\sin^2\vartheta\right)\nabla^2\right]\frac{\delta n_e}{n_0}=\frac{e^2}{c^2m_e^2}\nabla^2(\mathbf{A}_L\cdot\mathbf{A}_s), \tag{24}$$

where $\mathbf{A}_L$ and $\mathbf{A}_s$ are the vector potentials of the large-amplitude pump and the scattered light waves, δn_e is a small density perturbation in the plasma, n_0 is the plasma ambient density, and $\mu=v_{\rm os}/v_{\rm th}$. By means of the conventional method, for the backscatter and sidescatter ($0\leqslant\vartheta<\pi/4$), the dispersion relation can be derived as

$$(\omega^2-\eta^2\omega_{ek}^2)[(\omega-\omega_L)^2-c^2(\mathbf{k}-\mathbf{k}_L)^2-\omega_{pe}^2]=\frac{1}{4}k^2v_{\rm os}\omega_{pe}^2, \tag{25}$$

where, for $\omega_{pe}^2\gg 3k^2v_{\rm th}^2$,

$$\eta^2=1+\frac{3}{2}\left(\frac{v_{\rm os}}{c}\right)^2\left(\frac{\omega_L}{\omega_{pe}}\right)^2\sin^2(2\vartheta) \tag{26}$$

and the wave-number matching condition $k\simeq 2k_L\cos\vartheta$ is used. It needs to be indicated that, in order to keep in line with the usual treatment, the angle θ in Eq. (20) has been translated into ϑ, which represents the angle between the plasma wave vector and the laser propagation direction. One can see that the original wave frequency ω_{ek} is replaced by $\omega_c=\eta\omega_{ek}$, determined by Eq. (20). Taking $\omega=\omega_c+i\gamma$, the maximum growth occurs when the scattered light wave is also resonant. The relevant growth rate, in this case, can be obtained

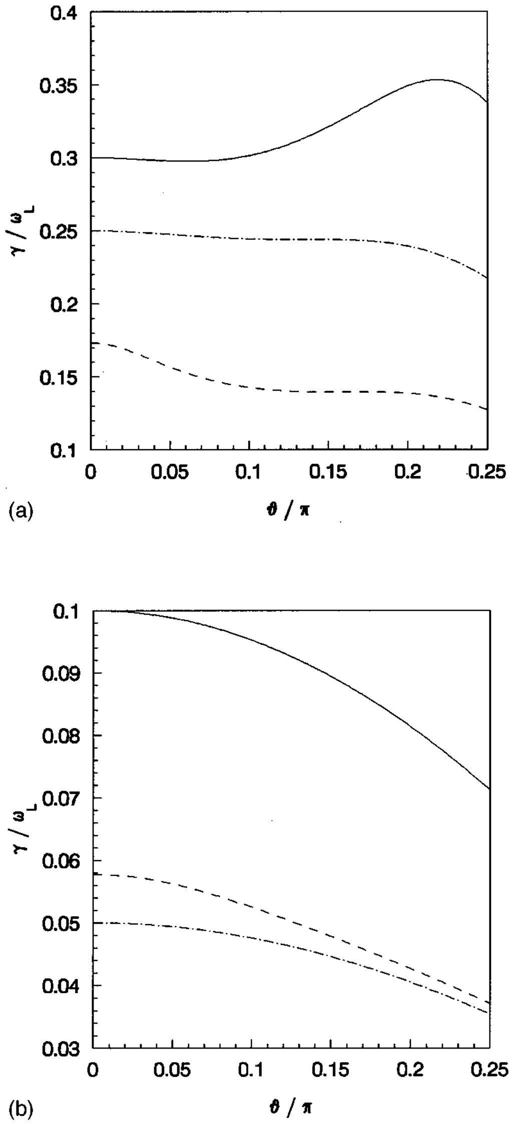

FIG. 7. Growth rate of the Raman instability vs the scattering angle ϑ. (a) For $v_{\rm os}/c=0.6$, the solid line is at the 1/4 critical density and the dashed line is at the 1/16 critical density. For $v_{\rm os}/c=0.5$, the dash-dotted line is at the 1/4 critical density. (b) For $v_{\rm os}/c=0.2$, the solid line is at the 1/4 critical density and the dashed line is at the 1/16 critical density. For $v_{\rm os}/c=0.1$, the dash-dotted line is at the 1/4 critical density.

$$\gamma=0.5\frac{v_{\rm os}}{c}\left[\frac{\omega_{pe}/\omega_L}{\eta(1-\eta\omega_{pe}/\omega_L)}\right]^{1/2}\omega_L\cos\vartheta. \tag{27}$$

In comparison with Eq. (23), we can find that one factor η appears, except for the scattering angle ϑ is included in Eq. (27). When $\vartheta=0$ it returns to Eq. (23), which gives the backscattering growth rate. The unstable mode wave number k that satisfies the scattered light resonant condition is

$$k=k_0\pm\frac{\omega_L}{c}\left[\left(1-\frac{\omega_{pe}^2}{\omega_L^2}\right)\cos^2\vartheta+\eta^2\frac{\omega_{pe}^2}{\omega_L^2}-2\eta\frac{\omega_{pe}}{\omega_L}\right]^{1/2}. \tag{28}$$

The growth rate as a function of the angle ϑ is plotted in Fig. 7. One can see that, at the plasma electron density $n_e=\frac{1}{4}n_{\rm cr}$, where $n_{\rm cr}$ is the critical density, the growth rate is gradually reduced along with angle ϑ when $v_{\rm os}/c\leqslant 0.5$. That is to say, the backscattering ($k\simeq 2k_L$) growth rate is larger than that of sidescattering in the strength region of $v_{\rm os}/c\leqslant 0.5$, which is in agreement with the usual result. When the strength value is increased to $v_{\rm os}/c\geqslant 0.6$ a very interesting change results that the sidescattering growth rate is larger than the backscattering one as $\vartheta\geqslant\pi/10$, which is different from that obtained before. At the density $n_e=\frac{1}{16}n_{\rm cr}$, the sidescattered growth rate is lower than the backscattered one. From Fig. 7 we can arrive at the important conclusion that when the laser strength arrives at a certain value, the large-angle sidescatter is more significant that the back-scatter around the quarter critical density. It is worth indicating that in the approximation in Eq. (20) the only requirements are $\omega/(\omega_{pe}k\lambda_D)\gg v_{\rm os}/v_{\rm th}\sin\vartheta\sin(\mathbf{k}_L\cdot\mathbf{x}-\omega_L t)$ and $\frac{3}{2}k^2v_{\rm os}^2\cos^2\vartheta\ll\omega_{pe}^2$. Therefore, the above results are reasonable. In addition, the results given in this section are valid only in the pump linearity approximation. A more extensive discussion on this subject is planned to be presented in another work.

VII. CONCLUSION

In this work we have given a nonequilibrium distribution function of the electron plasma mathematically and obtained a dielectric function in an intense laser field by using an appropriate treatment method that linearized the relevant slowly varying quantities. We find numerically that, in the plasma interaction with an intense laser field, the negative imaginary part of the dielectric function does not follow a plasma instability. The most excited wave modes are damped by a large Landau damping as $\omega/\omega_p\geqslant 2$ and the excitation properties are distinct due to different angles of the wave vector deviating from the laser field. The parameter $k\lambda_D$ region where the excitation occurs is presented for the diverse strength values of laser field. For $k\lambda_D\ll 1(>v_{\rm th}/v_{\rm os})$, a significant dispersion relation with the laser field correction in different directions of the wave vector is obtained. In this case, the frequencies of the excited wave modes have a notable increase for the small-angle oscillations and this increase is directly related to the square of the electron quiver velocity in laser field. We find further that the screening effect is anisotropic under an intense laser field. The screening length increases with the intensification of the laser strength and the screening effect disappears in the direction parallel to the laser field when the strength increases to a critical value $v_{\rm os}/v_{\rm th}=4.0$. Within a large-angle domain of the wave vector deviated from the direction of the laser field (e.g., $0\leqslant\theta\leqslant 0.3\pi$) the screening is lost when the strength is raised to a certain value (e.g., $v_{\rm os}/v_{\rm th}=6.0$). Furthermore, we indicate that in the physics of Coulomb scattering, replacing the maximum impact parameter with the Debye length may be unsuitable in the presence of an intense laser field if the mentioned screening effects can occur in the interaction time ω_{pe}^{-1}. Finally, we investigate the effects of these results on SRS instability. It is found that, for the strength value $v_{\rm os}/c>0.5$, the sidescatter starts out to become important because of the change of the dispersion relation in the electron plasma wave. In particular, when $v_{os}/c \geqslant 0.6$ and $\vartheta \geqslant \pi/10$ the instability growth rate is larger than that of the backscattering instability around the quarter critical density.

In the interaction of a plasma with an intense laser field, the electron quiver motion results in the anisotropy of the density fluctuations; however, it also results in the aeolotropism of the nonlinear coupling and the plasma response properties discussed in this paper. Only in laser propagating direction, where parameter $\xi = (\omega/k)\sqrt{m_e/T_e}$, do all the properties return to that of the Maxwellian plasma.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation of China, the National High Technology Committ of Laser, and Science Foundation of China Academy of Engineering Physics.

[1] P. Maine, D. Strickland, P. Bado, M. Pessot, and G. Mourou, IEEE J. Quantum Electron. **24**, 398 (1988).

[2] D. Strickland and G. Mourou, Opt. Commum. **56**, 216 (1985).

[3] M. D. Perry, F. G. Patterson, and J. Weston, Opt. Lett. **15**, 1400 (1990).

[4] P. Sprangle, E. Esarey, A. Ting, and G. Joyce, Appl. Phys. Lett. **53**, 2146 (1988).

[5] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. Lett. **64**, 2011 (1990); Phys. Rev. A **41**, 4463 (1990).

[6] C. Max, J. Arons, and A. B. Langdon, Phys. Rev. Lett. **33**, 209 (1974).

[7] W. B. Mori, C. Joshi, J. M. Dawson, D. W. Forslund, and I. M. Kindel, Phys. Rev. Lett. **60**, 1298 (1988).

[8] S. C. Wilks, J. M. Dawson, W. B. Mori, T. Katsouleas, and M. E. Jones, Phys. Rev. Lett. **62**, 2600 (1989).

[9] E. Esarey, A. Ting, and P. Sprangle, Phys. Rev. A **42**, 3526 (1990).

[10] E. Esarey, G. Joyce, and P. Sprangle, Phys. Rev. A **44**, 3908 (1991).

[11] C. Decker, W. B. Mori, J. M. Dawson, and T. Katsouleas, Phys. Plasmas **1**, 4043 (1994).

[12] A. Modenna *et al.*, Nature **377**, 606 (1995).

[13] W. B. Mori, C. D. Decker, D. E. Hinkel and T. Katsouleas, Phys. Rev. Lett. **72**, 1482 (1994).

[14] T. M. Antonsen, Jr. and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).

[15] C. Decker, W. B. Mori, K. C. Tzeng, and T. Katsouleas, Phys. Plasmas **3**, 2047 (1996).

[16] C. Rousseaux *et al.*, Phys. Rev. Lett. **74**, 4655 (1995).

[17] C. A. Coverdale *et al.*, Phys. Rev. Lett. **74**, 4659 (1995).

[18] X. Liu and D. Umstadter, Phys. Rev. Lett. **69**, 1935 (1992).

[19] J. C. Solem, T. S. Luk, K. Boyer, and C. K. Rhodes, IEEE J. Quantum Electron. **25**, 2423 (1989).

[20] D. C. Eder *et al.*, Phys. Plasmas **1**, 1744 (1994).

[21] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[22] P. Sprangle, E. Esarey, J. Krall, and G. Joyce, Phys. Rev. Lett. **69**, 2200 (1992).

[23] K. Morawetz and A. P. Jauho, Phys. Rev. E **50**, 474 (1994).

[24] S. Rand, Phys. Rev. **136**, B231 (1964).

[25] R. D. Jones and K. Lee, Phys. Fluids B **25**, 2307 (1982).

[26] S. C. Rae and K. Burnett, Phys. Fluids B **2**, 1015 (1990).

[27] E. A. Jackson, Phys. Rev. **153**, 235 (1967).

[28] K. J. Harker, Int. J. Electron. **32**, 297 (1972).

[29] J. N. Leboeuf and T. W. Johnston, Can. J. Phys. **53**, 2387 (1975).

[30] See, e.g., R. C. Davidson, in *Basic Plasma Physics I*, Handbook of Plasma Physics Vol. 1, edited by M. N. Rosenbluth and R. Z. Sagdeev (North-Holland, New York, 1983), p. 519.

[31] V. P. Silin, Zh. Éksp. Teor. Fiz. **48**, 1679 (1965) [Sov. Phys. JETP **21**, 1127 (1965)]; Usp. Fiz. Nauk. **108**, 625 (1972) [Sov. Phys. Usp. 1973 (1973)].

[32] J. R. Sanmartin, Phys. Plasmas **13**, 1353 (1970).

[33] J. E. Bernard, H. A. Baldis, D. M. Villeneuve, and A. B. Langdon, Phys. Rev. Lett. **58**, 1644 (1987).

[34] S. C. Wilks, W. L. Kruer, K. Estabrook, and A. B. Langdon, Phys. Fluids B **4**, 2794 (1992).

[35] W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, New York, 1988).